Shift Operator for Nonabelian Lattice Current Algebra

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Abstract. The shift operator for a quantum lattice current algebra associated with $sl(2)$ is produced in the form of product of local factors. This gives a natural deformation of the Sugawara construction for discrete space-time.

Introduction

The Current Algebra provides the chiral dynamical variables for a generic conformal field theory model called WZNW model. Its lattice analogue, due to Semenov-Tian-Shansky, proved useful for elucidating the quantum group structure in this model [AFSV]. In the subsequent papers [AFS, FG, BC, AFFS] some general properties of this algebra and its representations were discussed. However, these considerations covered kinematical aspects of the lattice model while such a basic dynamical object as the hamiltonian density remained unavailable. Here we address this problem making use of our experience in a simpler abelian case [FV93, V95]. Following the general philosophy worked out in these papers we construct a spatial translation operator W which simultaneously generates the temporal shift. We find W to be a product of local factors over the lattice. This may be regarded as a multiplicative analogue of the Sugawara construction.

For simplicity we confine ourselves to the simplest case of the $sl(2)$ algebra. In Section 1 we recall the basic facts about the current algebra in its classical continuous form. Then we embed the Sugawara hamiltonian into the hierarchies of conservation laws of two major integrable models which are mKdV and NLS equations [FT]. To make a smoother transition to the quantum case we present in Section 3 the classical lattice deformation of the current algebra. In particular, we produce relevant integrable hierarchies. The quantum case is treated in Section 4.

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1   Classical Model

The generators $j^a(x)$ of current algebra are associated with a given simple Lie algebra g with index a labeling the linear basis in g and the variable x running through the unit circle. Let f_c^{ab} and K^{ab} be the structure constants and the Killing tensor of g. The defining Poisson bracket is

$$\{j^a(x), j^b(y)\} = \gamma f_c^{ab} j^c(x)\delta(x-y) + \gamma K^{ab}\delta'(x-y) \ .$$

The real 'coupling constant' γ is irrelevant in classical case but comes into play under quantization.

The hamiltonian

$$H = \frac{1}{2\gamma} \int_0^{2\pi} K_{ab} j^a(x) j^b(x) dx$$

leads to a free equation of motion

$$\partial_t j^a(x) = \{H, j^a(x)\} = \partial_x j^a(x)$$

which reflects the conformal invariance in hyperbolic language. The hamiltonian density

$$T(x) = K_{ab} j^a(x) j^b(x)$$

is quadratic in the generators and is often referred to as the Sugawara construction. In this paper we shall consider g to be a real form $sl(2, R)$ of the algebra $sl(2)$.

Thus, a takes the values $3, +, -$ and all the functions j^a are real. The Poisson bracket is given by

$$\{j^3(x), j^3(y)\} = \gamma\delta'(x-y)$$

$$\{j^3(x), j^\pm(y)\} = \pm\gamma j^\pm\delta(x-y)$$

$$\{j^+(x), j^-(y)\} = 2\gamma(j^3\delta(x-y) + \delta'(x-y))$$

$$\{j^\pm(x), j^\pm(y)\} = 0$$

and

$$T = (j^3)^2 + j^+ j^-.$$

It is also useful to combine the currents into a 2 by 2 matrix

$$J = \begin{pmatrix} j^3 & j^- \\ j^+ & -j^3 \end{pmatrix} = \sum j^a \sigma_a$$

and write the Poisson bracket in the form

$$\{\overset{1}{J}(x), \overset{2}{J}(y)\} = \frac{\gamma}{2}[C, \overset{1}{J}(x) - \overset{2}{J}(y)]\delta(x-y) + \gamma C\delta'(x-y)$$

where the standard notation of the R-matrix formalism is employed [FT], and C is the Casimir element.

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2 Separation of Variables and Yang-Baxterization

The above bracket and hamiltonian allow for separation of variables. Indeed, one may put the matrix J into the form

$$J = \Omega \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix} \Omega^{-1} + \partial_x \Omega \Omega^{-1}$$

with a diagonal matrix Ω solving the equation

$$\partial_x \Omega = j^3 \sigma_3 \Omega.$$

The Poisson bracket for the new set of dynamical variables $j \equiv j^3$, p, q proves to be

$$\{p(x), p(y)\} = -2\gamma \text{sign}(x - y)p(x)p(y)$$

$$\{q(x), q(y)\} = -2\gamma \text{sign}(x - y)q(x)q(y)$$

$$\{p(x), q(y)\} = 2\gamma(\text{sign}(x - y)p(x)p(y) + \delta'(x - y))$$

$$\{j(x), j(y)\} = \gamma \delta'(x - y)$$

$$\{j(x), p(y)\} = \{j(x), q(y)\} = 0.$$

while the hamiltonian density becomes

$$T = j^2 + pq.$$

Thus, the pair p, q completely separates from j. The p-q bracket is known to belong to the hierarchy of Poisson structures associated with the NLS equation while the density pq is a member (the momentum density) of the corresponding family of densities of local conservation laws [FT]. On the other hand, the j-bracket and the density j^2 come from the hierarchy of the mKdV equation. Thus, we see where $sl(2)$ current algebra and Sugawara hamiltonian fit into the general pattern of Soliton Theory:

$$H_{\text{WZNW}} = P_{\text{mKdV}} + P_{\text{NLS}}.$$

This will prove useful for our approach to quantization.

The lattice formalism for the mKdV part, which is nothing but the abelian current algebra, was already developed in [FV93, V92, V95]. In this paper we perform a similar treatment of the NLS part.

In Soliton Theory the densities of conservation laws come from the asymptotic expansion of the trace of the monodromy matrix of the auxiliary linear problem. For the NLS equation this problem reads

$$\left(\partial_x + \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix} + \lambda\sigma_3\right)\Psi = 0.$$

The matrix Ω being diagonal, this auxiliary problem is gauge equivalent to

$$\left(\partial_x + J + \lambda\sigma_3\right)\Psi = 0.$$

Thus, we see that NLS part P_{NLS} of the Sugawara hamiltonian is provided by 'Yang-Baxterization' of the current

$$J \rightsquigarrow J + \lambda\sigma_3.$$

In the next Section we shall do the same on the lattice.

3   Lattice Model

We discretize the circle introducing the spacial variable taking integer values running from 1 to N. The real dynamical variables will be denoted by $\alpha_{n+\frac{1}{2}}, \beta_n$ with integer n; it is understood that

$$\alpha_{n+N+\frac{1}{2}} = \alpha_{n+\frac{1}{2}}$$

$$\beta_{n+N} = \beta_n.$$

One may say that integers label vertices while half-integers stand for edges. Or vice versa. One reason for using different notations for dynamical variables with integer and half-integer subscripts is mere convenience which becomes evident when the Poisson bracket is displayed:

$$\{\alpha_{n-\frac{1}{2}}, \alpha_{n+\frac{1}{2}}\} = -2\gamma\alpha_{n-\frac{1}{2}}\alpha_{n+\frac{1}{2}}$$

$$\{\alpha_{n-\frac{1}{2}}, \beta_n\} = -2\gamma\alpha_{n-\frac{1}{2}}\beta_n$$

$$\{\beta_n, \alpha_{n+\frac{1}{2}}\} = -2\gamma\beta_n\alpha_{n+\frac{1}{2}}$$

$$\{\beta_{n-1}, \beta_n\} = 2\gamma\alpha_{n-\frac{1}{2}} \quad .$$

All brackets not listed are zero. It is clear that every variable has nontrivial brackets only with the two neighbours in either direction.

One can recognise here the so called Flaschka variables for the Toda model. However, the hierarchy we will deal with is different from that of the Toda equations.

To see what this lot has to do with the Current Algebra we arrange dynamical variables in two matrices

$$B_{2n} = \begin{pmatrix} \alpha_{2n+\frac{1}{2}}^{-\frac{1}{2}} & 0 \\ 0 & \alpha_{2n+\frac{1}{2}}^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1 & \beta_{2n} \\ 0 & 1 \end{pmatrix}$$

$$C_{2n-1} = \begin{pmatrix} \alpha_{2n-\frac{1}{2}}^{\frac{1}{2}} & 0 \\ 0 & \alpha_{2n-\frac{1}{2}}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \beta_{2n-1} & 1 \end{pmatrix}.$$

The Poisson relations for them

$$\{\overset{1}{B}_{2n}, \overset{2}{B}_{2n}\} = \gamma[r_{12}, \overset{1}{B}_{2n}\overset{2}{B}_{2n}]$$

$$\{\overset{1}{C}_{2n-1}, \overset{2}{C}_{2n-1}\} = \gamma[r_{21}, \overset{1}{C}_{2n-1}\overset{2}{C}_{2n-1}]$$

$$\{\overset{1}{B}_{2n}, \overset{2}{C}_{2n-1}\} = \gamma\overset{1}{B}_{2n}r_{12}\overset{2}{C}_{2n-1}$$

$$\{\overset{1}{C}_{2n+1}, \overset{2}{B}_{2n}\} = \gamma\overset{1}{C}_{2n+1}r_{21}\overset{2}{B}_{2n}$$

employ the major ingredient of q-deformations, namely the classical r-matrices

$$r_{12} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 2 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$r_{21} = P_{12}r_{12}P_{12},$$

where P is a permutation.

The product

$$J_n = B_{2n}C_{2n-1}$$

satisfies the Poisson brackets

$$\{\overset{1}{J}_n, \overset{2}{J}_n\} = \gamma(r_{12}\overset{1}{J}_n\overset{2}{J}_{2n} - \overset{1}{J}_n\overset{2}{J}_{2n}r_{21})$$

$$\{\overset{1}{J}_{n+1}, \overset{2}{J}_n\} = \gamma\overset{1}{J}_{n+1}r_{21}\overset{2}{J}_n$$

which turn into the Current Algebra in the continuum limit

$$J_n \sim I + \Delta J(x).$$

This is what usually is called the Lattice Current Algebra. However, it is not clear whether one gains anything reducing B-C-algebra to the J-one. This time we prefer to deal with somewhat more transparent B-C-algebra but we could do with the J-one instead.

To produce relevant conservation laws we introduce the so called 'transfer-matrix'

$$t(\omega) = \mathrm{tr}\overset{\leftarrow}{\prod_n}\xi^{\sigma_3}B_{2n}\eta^{-\sigma_3}C_{2n-1}$$

with 'spectral parameter' ω entering in ξ and η in such a way that

$$\xi^2 + \eta^2 = 2$$

$$\frac{\xi}{\eta} = \omega.$$

It turns out that

(i) $t(\omega)$ is a Poisson commuting family:

$$\{t(\omega), t(\omega')\} = 0,$$

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(ii) in the continuous limit it turns into the trace of monodromy matrix of the continuous auxiliary linear problem of Section 1 provided

$$\omega \sim 1 + \Delta\lambda,$$

(iii) it is a power series in ω

$$t(\omega) = \sum_{-N/2}^{N/2} h_k \omega^{2k}$$

with

$$h_{N/2} = \prod_n \left(2 + \frac{\beta_{2n+1}\beta_{2n}}{\alpha_{2n+\frac{1}{2}}}\right)$$

$$h_{-N/2} = \prod_n \left(2 + \frac{\beta_{2n}\beta_{2n-1}}{\alpha_{2n-\frac{1}{2}}}\right).$$

(ii) is obvious, (iii) is almost so, (i) can be verified along the guidelines of [FM]. We shall not go into further details because the model in question actually belongs to the same hierarchy as the Ablowitz-Ladik's model [SV].

In the continuous limit we have

$$H = \log h_{N/2} + \log h_{-N/2} \sim \Delta \int_0^{2\pi} J^+ J^- dx = \Delta P_{\text{NLS}}$$

as should be expected.

We have obtained the hamiltonian of the classical lattice model which plays the role of the NLS part of the Sugawara construction for the lattice current algebra. The corresponding mKdV part can be found in [V92]. However, the equations of motion produced by these hamiltonians are quite complicated and turn into simple free equations only in the continuous limit. It was realized in [FV93] that the discrete time equation

$$J_n(t + \Delta) = J_{n+1}(t)$$

is a better option. In other words, the discretizing of space should be accompanied by the discretizing of time. The last equation is especially transparent in the quantum theory where the spacial shift operator W such that

$$W^{-1} J_n W = J_{n+1}$$

is taken to define the time shift as well

$$J_n(t + \Delta) = W^{-1} J_n(t) W$$

$$J_n(0) = J_n.$$

We shall find this operator in the next Section. The expression for the classical lattice hamiltonian will prove to be a useful hint in our search.

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4   Shift Operator

The quantum lattice current algebra inherits the notation α-β for generators together with the way they are enumerated while the Poisson relations turn into their most natural quantum counterparts

$$\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}} = q^2\alpha_{n-\frac{1}{2}}\alpha_{n+\frac{1}{2}}$$

$$\beta_n\alpha_{n-\frac{1}{2}} = q^2\alpha_{n-\frac{1}{2}}\beta_n$$

$$\alpha_{n+\frac{1}{2}}\beta_n = q^2\beta_n\alpha_{n+\frac{1}{2}}$$

$$[\beta_{n-1},\beta_n] = (q - q^{-1})\alpha_{n-\frac{1}{2}} \quad ,$$

with the deformation parameter q combining the coupling constant γ and the Planck constant $\hbar$ in the usual way

$$q = e^{i\hbar\gamma}.$$

The consistency of these commutation relations becomes more apparent as soon as one rewrites them in R-matrix form

$$R_{12}\overset{1}{B}_{2n}\overset{2}{B}_{2n} = \overset{2}{B}_{2n}\overset{1}{B}_{2n}R_{12}$$

$$R_{21}\overset{1}{C}_{2n-1}\overset{2}{C}_{2n-1} = \overset{2}{C}_{2n-1}\overset{1}{C}_{2n-1}R_{21}$$

$$\overset{2}{C}_{2n-1}\overset{1}{B}_{2n} = \overset{1}{B}_{2n}R_{12}\overset{2}{C}_{2n-1}$$

$$\overset{2}{B}_{2n}\overset{1}{C}_{2n+1} = \overset{1}{C}_{2n+1}R_{21}\overset{2}{B}_{2n}$$

where matrices B,C are built of α,β's in literally the same way as in the classical case of Section 2. The R-matrix involved is the $sl(2)$ one

$$R_{12} = \begin{pmatrix} q^{\frac{1}{2}} & & & \\ & q^{-\frac{1}{2}} & q^{\frac{1}{2}} - q^{-\frac{3}{2}} & \\ & & q^{-\frac{1}{2}} & \\ & & & q^{\frac{1}{2}} \end{pmatrix}$$

and it is needless to say that the associativity of the B-C-algebra is due to the Yang-Baxter equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

fulfilled by R.

The way variables separate in the continuous model and the belief that integrals of local densities on the lattice turn into products of local factors suggest that the shift operator

$$\alpha_{n-\frac{1}{2}}W = W\alpha_{n+\frac{3}{2}}$$

$$\beta_{n-1}W = W\beta_{n+1}$$

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should decompose into a product of two commuting factors

$$W = UV = VU$$

of the form

$$U = \overleftarrow{\prod} \theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})$$

$$V = \overleftarrow{\prod} \sigma(t_{n-\frac{1}{2}})$$

$$t_{n-\frac{1}{2}} = q + q^2\beta_n\alpha_{n-\frac{1}{2}}^{-1}\beta_{n-1}.$$

It only remains to find suitable functions θ and σ. It is shown in Appendix B that the functions doing the job are

$$\theta(z) = \exp \frac{(\log(-z))^2}{4\log q}$$

and

$$\sigma(z) = \exp \frac{1}{4} \int_{-\infty}^{\infty} \frac{z^{i\zeta}}{\sinh(\pi\zeta)\sinh(\gamma\hbar\zeta)} \frac{d\zeta}{\zeta}$$

where the contour of integration rounds the singularity at $\zeta = 0$ from above.

Let us conclude with some remarks. It must be said that although the solution looks like a straightforward remake of the one for the $U(1)$ case [FV93] it is not. The logics of the abelian case would rather favour another local decomposition for the shift operator, with functions $\theta(-z)$ and $\sigma(-z)$ instead of $\theta(z)$ and $\sigma(z)$. Of course, the minus in the argument of σ is a lot more important than it may look. In particular, the r.h.s. of the functional equation fulfilled by $\sigma(z)$ (see Appendix) comes directly from the density of the classical lattice hamiltonian

$$\frac{1}{1+t} \sim \frac{1}{2+\beta\beta/\alpha}.$$

This correspondence principle plays a major role in the detailed study of the classical limit which will be presented elsewhere.

As we said, one could do with the J-picture from the very beginning. This would eventually lead to the following decomposition

$$W = \overleftarrow{\prod} \sigma(t_{2n+\frac{1}{2}})\theta(q\alpha_{2n+\frac{1}{2}}\alpha_{2n-\frac{1}{2}}^{-1})\sigma(t_{2n-\frac{1}{2}})$$

for the J-shifting operator

$$J_n W = W J_{n+1}.$$

It is easy to check that all entries of this decomposition do belong to the J-subalgebra of the B-C-algebra. However, it is not that apparent why the reduction B-$C \rightarrow J$ should eliminate some factors in W and modify some of remaining ones. Subtle things like central elements are partly responsible for this. Which in turn has a lot to do with another important issue which we deliberately left aside. Indeed, we did not pay proper attention to the periodic boundary condition. What was then meant by ordered products over the lattice? For the answer to this question we refer the reader to [V95].

Appendix. Full Pull

In this paper, as well as on a few earlier occasions [FV93, FV94, F, FV95], we relied on certain 'q-special-functions' satisfying functional equations of the form

$$\frac{f(qz)}{f(q^{-1}z)} = d(z).$$

In particular, the functions θ and σ of the last Section were given by $d(z) = z$ and $d(z) = \frac{1}{1+z}$ respectively. Although the issues of the solvability and the good choice of solution are important in their own right, for practical purposes (read formal computations) one seldom needs anything else than just the equation itself. Let us show how it goes in our case.

Due to the locality of commutation relations the equations

$$\alpha_{n-\frac{1}{2}}W = W\alpha_{n+\frac{3}{2}}$$

$$\beta_{n-1}W = W\beta_{n+1}$$

easily reduce to

$$\alpha_{n-\frac{1}{2}}\theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1}) = \theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\alpha_{n+\frac{3}{2}}.$$

and

$$\beta_{n-1}\sigma(t_{n+\frac{1}{2}})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\sigma(t_{n-\frac{1}{2}}) = \sigma(t_{n+\frac{1}{2}})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\sigma(t_{n-\frac{1}{2}})\beta_{n+1}.$$

The mere form of the functional equation suggests that any computation ought to be a sequence of just two elementary steps:

(i) $w f(z) = f(z)\, w$ if $wz = zw$,

(ii) $x f(y) = f(y)\, d(qy)x$ if x, y make a Weyl pair: $xy = q^2 yx$.

Since any two of the α's either commute or make a Weyl pair, the first translation comes easy:

$$\alpha_{n-\frac{1}{2}}\theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})$$

$$= -q^2\theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1}\alpha_{n-\frac{1}{2}}\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})$$

$$= q^4\theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1}\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1}\alpha_{n-\frac{1}{2}}$$

$$= \theta(q\alpha_{n+\frac{3}{2}}\alpha_{n+\frac{1}{2}}^{-1})\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\alpha_{n+\frac{3}{2}}.$$

The second one is more tricky. We cannot pull β_{n-1} through $\sigma(t_{n+\frac{1}{2}})$ straight away because β_{n-1} and $t_{n+\frac{1}{2}}$ neither commute nor make a Weyl pair.

Nevertheless, we have a good supply of operators making 'good' pairs with both $t_{n-\frac{1}{2}}$ and $t_{n+\frac{1}{2}}$ which, by the way, between themselves are a q-oscillator*

$$qt_{n+\frac{1}{2}}t_{n-\frac{1}{2}} - q^{-1}t_{n-\frac{1}{2}}t_{n+\frac{1}{2}} = q - q^{-1}.$$

Among them are:

(i) all the α's
$$[\alpha, t] = 0,$$

(ii) the β which is 'between' them
$$t_{n-\frac{1}{2}}\beta_n = q^2\beta_n t_{n-\frac{1}{2}}$$
$$\beta_n t_{n+\frac{1}{2}} = q^2 t_{n+\frac{1}{2}}\beta_n,$$

(iii) another operator $c_n = q(t_{n+\frac{1}{2}}t_{n-\frac{1}{2}} - 1)$
$$t_{n-\frac{1}{2}}c_n = q^2 c_n t_{n-\frac{1}{2}}$$
$$c_n t_{n+\frac{1}{2}} = q^2 t_{n+\frac{1}{2}}c_n$$

which is a familiar satellite of q-oscillators.

So, we express β_{n-1} via 'good' operators

$$\beta_{n-1} = qt_{n+\frac{1}{2}}^{-1}c_n\beta_n^{-1}\alpha_{n-\frac{1}{2}} + q^2 t_{n+\frac{1}{2}}^{-1}\beta_n^{-1}\alpha_{n-\frac{1}{2}} - q\beta_n^{-1}\alpha_{n-\frac{1}{2}}$$

and get

$$\beta_{n-1}\sigma(t_{n+\frac{1}{2}})$$

$$= \sigma(t_{n+\frac{1}{2}})\left(qt_{n+\frac{1}{2}}^{-1}c_n\beta_n^{-1}\alpha_{n-\frac{1}{2}} + (1 + q^{-1}t_{n+\frac{1}{2}})(q^2 t_{n+\frac{1}{2}}^{-1}\beta_n^{-1}\alpha_{n-\frac{1}{2}} - q\beta_n^{-1}\alpha_{n-\frac{1}{2}})\right)$$

$$= \sigma(t_{n+\frac{1}{2}})\left((\beta_n^{-1}t_{n-\frac{1}{2}} - t_{n+\frac{1}{2}}\beta_n^{-1})\alpha_{n-\frac{1}{2}}\right).$$

Similarly,

$$\sigma(t_{n-\frac{1}{2}})\beta_{n+1} = \left(\alpha_{n+\frac{1}{2}}(-\beta_n^{-1}t_{n-\frac{1}{2}} + t_{n+\frac{1}{2}}\beta_n^{-1})\right)\sigma(t_{n-\frac{1}{2}}).$$

The rest

$$\left((\beta_n^{-1}t_{n-\frac{1}{2}} - t_{n+\frac{1}{2}}\beta_n^{-1})\alpha_{n-\frac{1}{2}}\right)\theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})$$

$$= \theta(q\alpha_{n+\frac{1}{2}}\alpha_{n-\frac{1}{2}}^{-1})\left(\alpha_{n+\frac{1}{2}}(-\beta_n^{-1}t_{n-\frac{1}{2}} + t_{n+\frac{1}{2}}\beta_n^{-1})\right)$$

is a variation on the same theme.

Unfortunately, all this looks like a hat-trick rather than conscious approach. A more systematic paper making better use of YangBaxterization is on the authors' agenda.

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*The remaining nontrivial commutation relations governing the algebra of t's, those for the neighbours twice removed, seldom participate in computations. Their role may be seen in taking care of the associativity of the algebra. Anyway, their explicit form can be found in [V92]. This algebra is sometimes referred to as the Lattice Virasoro Algebra for in a certain continuous limit, different from the one of the present paper, it turns into the Virasoro algebra with a nonzero central charge.

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